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Basic Algorithms for Digital Image Analysis: a course

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Lecture 7: Corner detection in greyscale images

- Corners in curves and images
- Importance of corners
  - in motion analysis
  - in shape perception and analysis
- Corner detection in greyscale images
  - Local structure matrix
  - KLT corner detector
  - Harris corner detector
  - Comparison of the two corner detectors

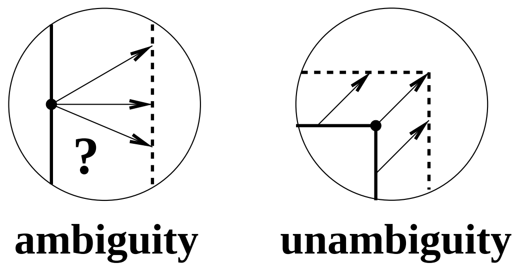
Corners in curves and images

A corner is a sharp turn of a contour.

- Corners are used in shape analysis and motion analysis
- Corners and other points of high curvature are dominant in **human perception of 2D shapes**
  - Shapes can be approximately reconstructed from their dominant points
- Two different operations but related operations are called **corner detection**:
  - Detection of corners in **digital curves**
    ⇒ This assumes **extracted contours**
  - Detection of corners in **greyscale images**
    ⇒ This does not assume extracted contours

This lecture deals with corner detection in greyscale images.

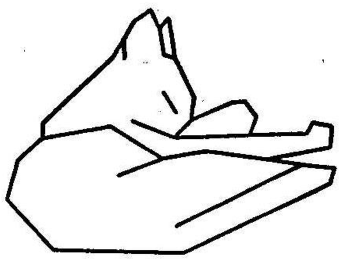
Importance of corners in motion analysis



The **aperture problem** *and the use of corners in motion analysis.*

- The displacement vectors are **ambiguous** at an edge.
- They are **unambiguous** at a corner.

Importance of corners in shape perception and analysis



The **Attneave's Cat**. *(Attneave, 1955)*

- The original smooth shape has been restored based on a small number of high curvature points.
- The cat is easy to recognise.

Corners, edges, and derivatives of intensity function

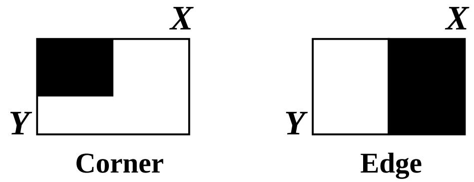
Diference between greyscale corners and edges:

- **Corners** are local image features characterised by locations where variations of intensity function $f(x, y)$ in both X **and** Y directions are high.

  ⇒ Both partial derivatives f_x and f_y are large

- **Edges** are locations where the variation of $f(x, y)$ in a certain direction is high, while the variation in the orthogonal direction is low.

  ⇒ In an edge oriented along the Y axis, f_x is large, while is f_y small



A corner and an edge.

Two selected corner detectors

Different corner detectors exist, but we will only consider two of them:

- The Kanade-Lucas-Tomasi (KLT) operator
- The Harris operator

Reasons:

- Most **frequently used**: Harris in Europe, KLT in US.
- Can select corners and other **interest points**.
- Have many application areas, for example:
  - motion tracking, stereo matching, image database retrieval
- Are relatively simple but still efficient and reliable.

The two operators are closely related and based on the **local structure matrix**.

The local structure matrix C_{str}

Definition of the local structure matrix (tensor):

$$C_{str} = w_G(r; \sigma) * \begin{bmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{bmatrix} \quad (1)$$

Explanation of the definition:

- The derivatives of the intensity function $f(x, y)$ are first calculated in each point.
  - If necessary, the image is smoothed before taking the derivatives
- Then, the entries of the matrix (f_x^2, etc.) are obtained.
- Finally, each of the entries is smoothed (integrated) by Gaussian filter $w_G(r; \sigma)$ of selected size σ.
  - Often, a simple box (averaging) filter is used instead of the Gaussian.

Properties of the local structure matrix

Denoting in (1) the smoothing by $\widehat{ff}$, we have

$$C_{str} = \begin{bmatrix} \widehat{f_x^2} & \widehat{f_x f_y} \\ \widehat{f_x f_y} & \widehat{f_y^2} \end{bmatrix}$$

The local structure matrix C_{str} is

- **Symmetric**

  ⇒ It can be **diagonalised** by rotation of the coordinate axes. The diagonal entries will be the two **eigenvalues** λ_1 and λ_2:

$$C_{str} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

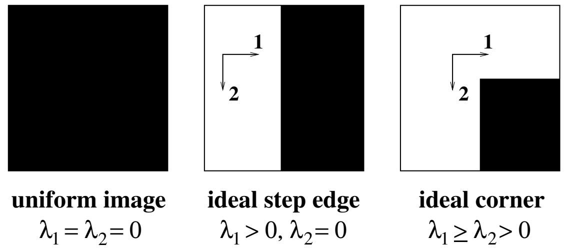
- **Positive semi-definite**

  ⇒ The eigenvalues are nonnegative. Assume $\lambda_1 \geq \lambda_2 \geq 0$.

Meaning of the eigenvalues of C_{str}

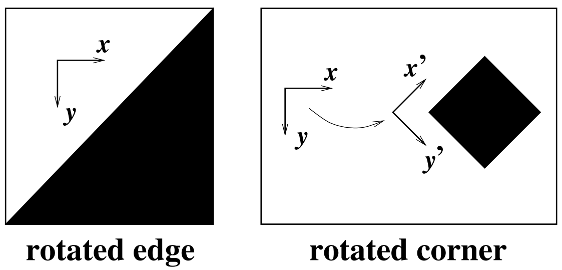
The geometric interpretation of λ_1 and λ_2:

- For a perfectly **uniform image**: $C_{str} = 0$ and $\lambda_1 = \lambda_2 = 0$.
- For a perfectly black-and-white **step edge**: $\lambda_1 > 0$, $\lambda_2 = 0$, where the eigenvector associated with λ_1 is orthogonal to the edge.
- For a **corner** of black square against a white background: $\lambda_1 \geq \lambda_2 > 0$.
  - The higher the contrast in that direction, the larger the eigenvalue



Basic observations:

- The eigenvectors encode edge directions, the eigenvectors edge magnitudes.
- A corner is identified by two strong edges ⇒ A corner is a location where the **smaller eigenvalue**, λ_2, is **large enough**.
- Diagonalisation of C_{str} means aligning the local coordinate axes with the two edge directions.
  - Remark: Setting a threshold on $\min(f_x, f_y)$ to find corners **would not work**!



Meaning of diagonalisation of the structure matrix.

The KLT corner detector has two parameters: the **threshold** on λ_2, denoted by λ_{thr}, and the linear size of a square **window** (neighbourhood) D.

Algorithm 1: The KLT Corner Detector

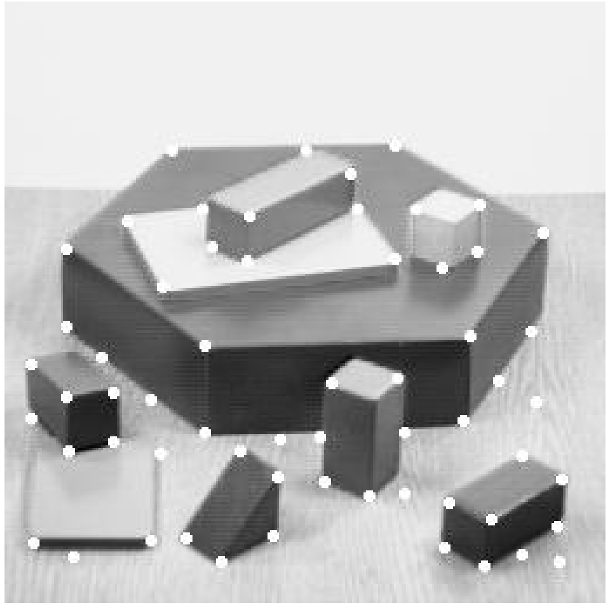
1. Compute f_x and f_y over the entire image $f(x, y)$.
2. For each image point p:
   (a) form the matrix C_{str} over a $D \times D$ neighbourhood of p;
   (b) compute λ_2, the smaller eigenvalue of C_{str};
   (c) if $\lambda_2 > \lambda_{thr}$, save p into a list, L.
3. Sort L in decreasing order of λ_2.
4. Scan the sorted list from top to bottom. For each current point, p, delete all points apperaring further in the list which belong to the neighbourhood of p.

The **output** is a list of feature points with the following properties:

- In these points, $\lambda_2 > \lambda_{thr}$.
- The D-neighbourhoods of these points do not overlap.

Selection of the parameters λ_{thr} and D:

- The threshold λ_{thr} can be estimated from the histogram of λ_2: usually, there is an obvious valley near zero.
  - Unfortunately, such valley is not **always** present
- There is no simple criterion for the window size D. Values between 2 and 10 are adequate in most practical cases.
  - For large D, the detected corner tends to move away from its actual position
  - Some corners which are close to each other may be lost



Example of corner detection by the KLT operator.

The Harris corner detector

The Harris corner detector (1988) appeared earlier than KLT. KLT is a **different interpretation** of the original Harris idea.

Harris defined a measure of **corner strength**:

$$H(x,y) = \det C_{str} - \alpha \left(\operatorname{trace} C_{str}\right)^2,$$

where α is a parameter and $H \geq 0$ if $0 \leq \alpha \leq 0.25$.

A **corner is detected** when

$$H(x,y) > H_{thr},$$

where H_{thr} is another parameter, a threshold on corner strength.

Similar to the KLT, the Harris corner detector uses D-neighbourhoods to discard weak corners in the neighbourhood of a strong corner.

Parameter of Harris operator and relation to KLT

Assume as before that $\lambda_1 \geq \lambda_2 \geq 0$. Introduce $\lambda_1 = \lambda$, $\lambda_2 = \kappa\lambda$, $0 \leq \kappa \leq 1$.

Using the relations between eigenvalues, determinant and trace of a matrix A

$$\det A = \prod_i \lambda_i$$

$$\operatorname{trace} A = \sum_i \lambda_i,$$

we obtain that

$$H = \lambda_1\lambda_2 - \alpha(\lambda_1+\lambda_2)^2 = \lambda^2\left(\kappa - \alpha(1+\kappa)^2\right)$$

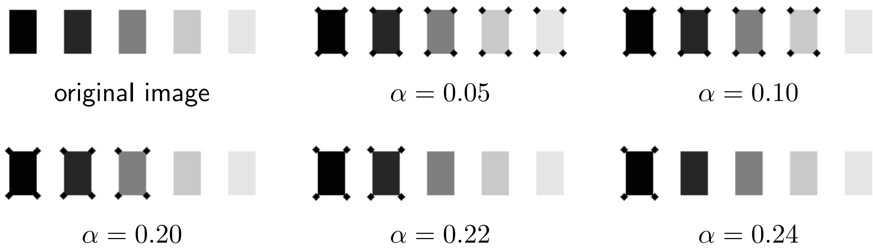
Assuming that $H \geq 0$, we have

$$0 \leq \alpha \leq \frac{\kappa}{(1+\kappa)^2} \leq 0.25 \quad \text{and, for small } \kappa,\ H \approx \lambda^2(\kappa - \alpha),\ \alpha \lesssim \kappa$$

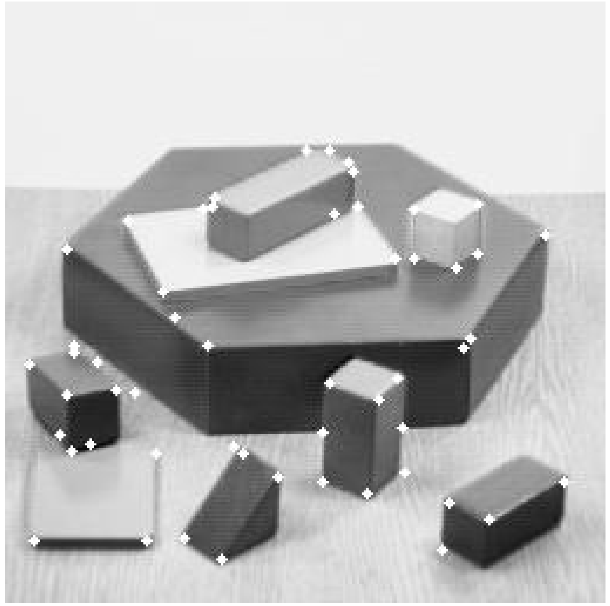
In the Harris operator, α plays a role similar to that of λ_{thr} in the KLT operator.

- Larger $\alpha \Rightarrow$ smaller $H \Rightarrow$ **less sensitive** detector: less corners detected.
- Smaller $\alpha \Rightarrow$ larger $H \Rightarrow$ **more sensitive** detector: more corners detected.

Usually, H_{thr} is set close to zero and fixed, while α is a variable parameter.

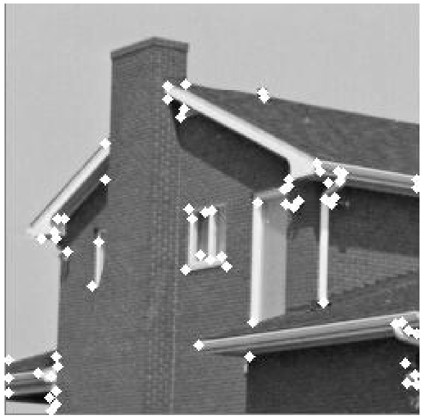


Corner detection by Harris operator: influence of α. ($H_{thr} = 0.$)



Example of corner detection by the Harris operator.

KLT 40 corners



Harris $\alpha = 0.2$

Comparison of the two operators.

Summary of corner detection

- The KLT and the Harris corner detectors are conceptually related.
  - Based on local structure matrix C_{str}
  - Search for points where variations in two orthogonal directions are large
- Difference between the two detectors:
  - KLT sets **explicit** threshold on the diagonalised C_{str}
  - Harris sets **implicit** threshold via corner magnitude $H(x, y)$
- The KLT detector
  - usually gives results which are closer to human perception of corners;
  - is often used for **motion tracking** in the wide-spread **KLT Tracker**.
- The Harris detector
  - provides good **repeatability** under varying rotation and illumination;
  - if often used in stereo matching and image database retrieval.
- Both operators may detect **interest points** other than corners.